

# MATH-4010 – Abstract Algebra

Professor M. Zuker – Spring 2008

Lectures:

- Jan 15: Sets, functions, equivalence relations and classes
- Jan 18: Binary relations & structures. Example: Representation of complex numbers as  $2 \times 2$  matrices for both  $+$  and  $\times$ . Homomorphism, isomorphism, associativity, commutativity, identity.
- Jan 22: Groups. Definition and left and right cancel rules. Abelian Group. Subgroup (subset  $H$  + closure) Bijections between a set and itself is a group under function composition. Special case:  $|S| = n$  (finite). Permutation group. Group tables and some simple examples.  $|G| = 4$  not resolved!
- Jan 25: Cyclic (sub)group generated by a single group element. Equivalence relation induced by a subgroup. Equivalence classes as cosets. Left and right cosets. Lagrange's Theorem:  $|H|$  divides  $|G|$ . Normal subgroup and how this allows one to define a natural group operation on the equivalence classes.
- Jan 29: Cyclic groups. Isomorphic to  $\mathbb{Z}$  or  $\mathbb{Z}_k$ . (Proved former if order is infinite.) Division algorithm, gcd and explicit computation of  $b_1$  and  $b_2$  such that  $a_1b_1 - a_2b_2 = 1$ , given  $\gcd(a_1, a_2) = 1$ .
- Feb 1: Finish with cyclic groups and subgroups. Subgroup diagrams. Generating sets. Permutations: Cayley's Theorem, Orbits, cycles.
- Feb 5: Transpositions, inversions, even/odd permutations. Alternating group. (If  $|G| = 2|H|$ ,  $H$  is normal).
- Feb 8: Direct products. Finitely generated Abelian groups.
- Feb 12: Classification of finitely generated Abelian groups.

Feb 15: Factor groups revisited. Subgroups: Kernel of a homomorphism (normal), range of a homomorphism, inverse image of co-domain

- (No class on Feb 19)

Feb22: Fundamental homomorphism theorem. Automorphisms.

Feb26: Simple groups, Center and commutator subgroups.

Feb29: Rings and Fields

Mar4: (continued)

Mar7: Integral Domains

- Spring Break: Mar 10 - 14

Mar18: (continued)

Mar21: Fermat's and Euler's Theorems.

Mar25: (continued)

Mar28: Polynomial rings

Apr1: (continued)

Apr4: Factoring polynomials over a field. Irreducible polynomials

Apr8: (continued)

Apr11: Non-commutative examples. Quaternions. Finite division rings.

Apr15: Ring homomorphisms and factor rings.

Apr18: Prime and maximal ideals

Apr22: (continued)

Apr25: Extension Fields

Apr29: (continued)

- May 5-9 Final Exams