Problem Set 2 Due Feb. 14. 2008

These problems are related to the "JavaScript" links on the course home page.

1. Adjacent transpositions (swaps).

   Using the `http://frontend.bioinfo.rpi.edu/zukerm/JavaScript/perm1.html` web page, determine the minimum number of adjacent transpositions, "l", "m" or "r" needed to achieve all possible permutations. What is the "worst case"? That is, which permutation in S_4 requires the maximum number?

   Generalize. What is the mimimum number of adjacent transpositions needed to achieve all possible permutations in S_n, $n > 4$? (Hint: Try using mathematical induction.)

2. Using the `http://frontend.bioinfo.rpi.edu/zukerm/JavaScript/perm2.html` web page, determine what subgroup of S_5 is generated by "l" and "r". Give your reasoning, not just an answer.

3. Using the `http://frontend.bioinfo.rpi.edu/zukerm/JavaScript/perm3.html` web page, show that D_5 is not commutative.

   Show that the subgroup $H = \{e, \tau\}$ is not normal. It suffices to find a single "left coset" of H that is not equal to the corresponding "right coset".

   Using the web form, for each left coset of H, what right coset of H equals the given left coset?

   What are the subgroups of D_5?

4. Using the `http://frontend.bioinfo.rpi.edu/zukerm/JavaScript/perm4.html` web page, on interlinking squares, answer the following questions.

   - Achieve a transposition of "1" and "2". What happens to the other vertices ("3", "4", "5" and "6")? What happens to the edges ("7", "8", "9", "10", "11", "12" and "13")?
   - Decompose "ba" into cycles. What is the order of the cyclic subgroup generated by "ba"?
   - Do the same as above for "Ba".
   - Can you transpose "1" and "2" while leaving the edges in place? Think "cycle length".