

## Problem Set 1 Due Jan. 25. 2008

1. 0: Problems 29, 30, 31, 32, 33 and 34
2. 1: Problems 7, 8, 9, 10 and 11
3. Extra problem.

Define a special class,  $G$ , of “linear rational functions” by

$$G = \left\{ f_{a,b,c,d}(x) = \frac{ax + b}{cx + d} \mid a, b, c, d \in \mathbb{Z}, ad - bc = 1 \right\}.$$

Define the binary operation,  $\star$ , by  $\circ$  (function composition). That is, if  $g, h \in G$ , then  $g \circ h(x) = g(h(x))$ .

- Show that  $G$  is closed under  $\star$ .
- We already know that function composition is associative.
- Show that  $G$  has an identity element. Do so by computing it. (It must be unique.)
- Show that each member of  $G$  has an inverse in  $G$ .
- $(G, \star)$  is a group. Is it commutative?

In the above problem, suppose  $a, b, c, d \in \mathbb{R}$  and that  $ad - bc \neq 0$ . Is  $G$  still a group?